Generation of exact solutions in cosmology on the basis of five-dimensional Projective Unified Field Theory

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Abstract.

A homogeneous and isotropic model of the Universe is considered in the framework of the five-dimensional Projective Unified Field Theory in which the gravitation is described by both space-time curvature and some hypothetical scalar field (σ-field). We propose a generation method for obtaining exact solutions. New exact Friedmann-like solutions for a dust model and inflationary solutions are found. It is shown that in the framework of exponential type inflation we obtain a natural explanation of why at present we do not observe σ-field effects or why these effects are negligible.

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1 Introduction

As is well known, the idea of a 5-dimensional unified field theory goes back to the works of Kaluza and Klein [1, 2]. The pioneers of the projective approach to this theory were Veblen and van Dantzig [3, 4]. Later this approach was further developed by many authors (see the corresponding references and a review of other higher-dimensional unified theories in [5, 6, 7]).

In this paper, a homogeneous and isotropic model of the Universe is considered in the framework of the 5-dimensional Projective Unified Field Theory (PUFT) developed by E. Schmutzer [8,9,10]. In PUFT, gravitation is described by both space-time curvature and some hypothetical scalar field (σ-field). To characterize the scalar field predicted in PUFT as a new fundamental phenomenon in Nature, Schmutzer introduced the notion "scalarism" (adjective: scalaric) by analogy with electromagnetism. A source of this "scalaric" field can be both the electromagnetic field and a new attribute of matter named by E. Schmutzer scalaric mass.

The PUFT is based on the postulated 5-dimensional Einstein-like field equations. By projecting them into the 4-dimensional space-time one can obtain the following 4-dimensional field equations (the cosmological term is omitted here) [9]:

$$R_{mn} - \frac{1}{2}\, g_{mn}\, R = \kappa_0 \left(E_{mn} + \Sigma_{mn} + \Theta_{mn}\right) \tag{1}$$

are the generalized gravitational field equations;

$$\text{a)}\quad H^{mn}{}_{;n} = \frac{4\pi}{c} j^m, \quad \text{b)}\quad B_{mn,k} + B_{km,n} + B_{nk,m} = 0, \quad \text{c)}\quad H_{mn} = e^{3\sigma} B_{mn} \tag{2}$$

are the generalized electromagnetic field equations;

$$\sigma^{,k}{}_{;k} = \kappa_0 \left(\frac{2}{3}\vartheta + \frac{1}{8\pi} B_{ik} H^{ik}\right) \tag{3}$$

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is the scalaric field equation. Here R_{mn} is the Ricci tensor,

$$E_{mn} = \frac{1}{4\pi}\left(B_{mk}H^{k}{}_{n} + \frac{1}{4}g_{mn}B_{ik}H^{ik}\right) \tag{4}$$

is the electromagnetic energy-momentum tensor,

$$\Sigma_{mn} = -\frac{3}{2\kappa_0}\left(\sigma_{,m}\sigma_{,n} - \frac{1}{2}g_{mn}\sigma_{,k}\sigma^{,k}\right) \tag{5}$$

is the scalaric energy-momentum tensor, Θ_{mn} is the energy-momentum tensor of the nongeometrized matter (substrate), H_{mn} and B_{mn} are the electromagnetic induction and the field strength tensor, respectively, j^k is the electric current density, ϑ is the scalaric substrate density, $\kappa_0 = 8\pi G/c^4$ is Einstein's gravitational constant (G is Newton's gravitational constant). Latin indices run from 1 to 4; the comma and semicolon denote partial and covariant derivatives, respectively; the signature of the metric is $+2$.

These field equations lead to the following generalized energy conservation law and continuity equation for electric current density:

$$\text{a)}\quad \Theta^{mn}{}_{;n} = -\frac{1}{c}B^{m}{}_{k}j^{k} + \vartheta\sigma^{,m}, \qquad \text{b)}\quad j^{m}{}_{;m} = 0. \tag{6}$$

It should be noted that recently E. Schmutzer offered a new variant of PUFT (see [10] and references therein) with slightly different 4-dimensional field equations as compared to the above-stated ones (one can find a detailed analysis of the geometric axiomatics of PUFT in [11]). Both variants are physically acceptable and deserve a comprehensive investigation. An analysis of Eqs. (1)–(5) shows that all the subsequent reasonings can be easily extended to the last version of PUFT.

2 Cosmological equations and generation of exact solutions

Let us consider a homogeneous and isotropic cosmological model with the Robertson-Walker line element in the well-known form:

$$\mathrm{d}s^2 = R^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\right] - c^2\mathrm{d}t^2, \tag{7}$$

where $R(t)$ is the scale factor and k takes the values 0 or ± 1. For an electrically neutral continuum, described by the energy-momentum tensor of a perfect fluid, Eqs. (1), (3) in the metric (7) lead to the following set of equations (the dot denotes a time derivative, ϱ is the mass density, p is the pressure):

$$\frac{\ddot{R}}{R} = -\frac{\kappa_0 c^2}{6}\left(\varrho c^2 + 3p\right) - \frac{1}{2}\dot{\sigma}^2, \tag{8}$$

$$\frac{\ddot{R}}{R} + \frac{2(\dot{R}^2 + kc^2)}{R^2} = \frac{\kappa_0 c^2}{2}\left(\varrho c^2 - p\right), \tag{9}$$

$$\ddot{\sigma} + 3\frac{\dot{R}}{R}\,\dot{\sigma} = -\frac{2}{3}\,\kappa_0 c^2\vartheta, \tag{10}$$

while the generalized energy conservation law (6) gives

$$\dot{\varrho} + 3\frac{\dot{R}}{R}\left(\varrho + \frac{p}{c^2}\right) = \frac{\vartheta}{c^2}\dot{\sigma}. \tag{11}$$

Eqs. (8) to (11) determine the dynamics of the cosmological model if the equations of state, i.e. $p = p(\varrho)$ and $\vartheta = \vartheta(\varrho)$, are given. The Friedmann model corresponds to the special case $\vartheta = 0$ and $\dot{\sigma} = 0$ of our model. Unfortunately, the above set of differential equations leads [12] to an Abel differential equation and till now was solved exactly only for some special cases [12, 13, 14, 15].

To obtain new exact solutions, let us consider the problem in a slightly different aspect. Firstly, we shall use the arbitrariness in the choice of an equation of state $\vartheta = \vartheta(\varrho)$, because the functional form $\vartheta(\varrho)$ is not

determined within the theory. Secondly, notice that above equations are not independent. For instance, it is possible to show that the last Eqs. (11) is a differential consequence of (8)–(10). Therefore on the further treatment we shall use only Eqs. (8)–(10). If the equation of state for ordinary matter is specified, $p = \nu\varrho c^2$ $(-1 \le \nu < 1)$, then Eqs. (8)–(10) can be reduced to a form where the functions $\sigma(t)$, $\varrho(t)$ and $\vartheta(t) \equiv \vartheta(\varrho(t))$ are expressed in terms of the function $R(t)$ and its derivatives:

$$\sigma(t) = \pm\sqrt{\frac{8}{3(1-\nu)}}\int\sqrt{-\frac{\ddot{R}}{R}-\frac{1+3\nu}{2}\,\frac{\dot{R}^2+kc^2}{R^2}}\,\mathrm{d}t+\sigma_0\ , \tag{12}$$

$$\varrho(t) = \frac{2}{(1-\nu)\kappa_0 c^4}\left[\frac{\ddot{R}}{R}+\frac{2(\dot{R}^2+kc^2)}{R^2}\right], \tag{13}$$

$$\vartheta(t) = -\frac{3}{2\kappa_0 c^2 R^3}\,\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{\sigma}R^3\right). \tag{14}$$

Here σ_0 is an integration constant and $\nu \ne 1$ (it is necessary to consider the case $\nu = 1$ separately). If we specify the time dependence of the scale factor, $R(t)$, we can find the corresponding functions $\sigma(t)$, $\varrho(t)$ and $\vartheta(t)$ which are necessary for this Universe evolution scenario. However, the choice of the dependence $R(t)$ is not free, because the natural requirement $\varrho \ge 0$, taking into account (12) and (13), gives the following restrictions on the choice of the scale factor:

$$-\frac{\ddot{R}}{R}-\frac{1+3\nu}{2}\,\frac{\dot{R}^2+kc^2}{R^2} \ge 0, \tag{15}$$

$$\frac{\ddot{R}}{R}+\frac{2(\dot{R}^2+kc^2)}{R^2} \ge 0. \tag{16}$$

It is obvious that the functions (13) and (14) determine a parametric dependence $\vartheta = \vartheta(\varrho)$, which in some cases can be reduced to an explicit form by eliminating t.

It should be noted that a similar method was proposed in [16, 17], where the idea that the shape of the potential of a self-interacting scalar field in standard inflationary models is not fixed, allows one to obtain new exact solutions for inflation (in this context see also [18, 19]). In [16] this approach was called the method of "fine turning of the potential".

3 Examples

3.1. Exact solutions for the dust model

To illustrate the above-stated method, we shall give a series of simple examples. We begin with a dust model ($p = 0$, or the constant $\nu = 0$). Let us consider a power-law behaviour of the scale factor:

$$R(t) = At^n \quad (1/3 \le n \le 2/3), \tag{17}$$

where A and n are positive constants and the limits for n were obtained from (15) and (16). It is easy to show that (17) satisfies (15) and (16) for $k = 0$ always and for $k = \pm 1$ only for a restricted period of time. Further, let us for simplicity restrict our consideration to the spatially-flat model ($k = 0$). With account of (17) Eqs. (12)–(14) allow us to find an exact solutions for $\sigma(t)$, $\dot{\sigma}(t)$, $\varrho(t)$ and $\vartheta(t)$:

$$\sigma(t) = \pm\, 2\,\sqrt{(2n-3n^2)/3}\,\ln t + \sigma_0\ , \tag{18}$$

$$\dot{\sigma}(t) = \pm\, 2\,\sqrt{(2n-3n^2)/3}\,\frac{1}{t}\ , \tag{19}$$

$$\varrho(t) = \frac{2(3n^2-n)}{\kappa_0 c^4}\,\frac{1}{t^2}\ , \tag{20}$$

$$\vartheta(t) = \mp \frac{(3n-1)\sqrt{3(2n-3n^2)}}{\kappa_0 c^2} \frac{1}{t^2} . \tag{21}$$

It is interesting to note that the case $n = 1/3$ corresponds to a universe with the scalar σ-field only (for this case the exact solutions were found earlier in [15]). The case $n = 2/3$ corresponds to the standard Einstein-de Sitter model of Friedmann's cosmology. Eqs. (20) and (21) allow one to obtain the equation of state $\vartheta = \vartheta(\varrho)$ in an explicit form:

$$\vartheta = \mp \frac{\sqrt{3(2n-3n^2)}}{2n} \varrho c^2 \quad (n \neq 1/3) , \tag{22}$$

and also to find the following range for ϑ:

$$0 \leq |\vartheta| < 3/2\varrho c^2. \tag{23}$$

Let us note the simplest consequences of this model for observational cosmology. The Hubble parameter and the age of the Universe are given by

$$H(t) \equiv \frac{\dot{R}}{R} = \frac{n}{t} ; \qquad t_0 = \frac{n}{H_0} , \quad \frac{1}{3H_0} \leq t_0 \leq \frac{2}{3H_0} , \tag{24}$$

where the subscript 0 denotes the present values. For the deceleration parameter q_0 we have

$$q_0 = q \equiv -\frac{\ddot{R}R}{\dot{R}^2} = \frac{1-n}{n} , \quad 1/2 \leq q_0 \leq 2 . \tag{25}$$

The parameter λ_0 (already introduced in [20]), characterizing the σ-field, is given by

$$\lambda_0 \equiv \frac{1}{H_0} \frac{\mathrm{d}\sigma(t_0)}{\mathrm{d}t} = \pm\, 2 \sqrt{\frac{1}{n}\left(\frac{2}{3} - n\right)} , \quad 0 \leq |\lambda_0| \leq 2 . \tag{26}$$

It should be noted that the parameter λ_0 is, in principle, a measurable quantity [20]. Also, for the flat model considered here, λ_0 and q_0 are not independent parameters: they are related by the identity $2q_0 - 3{\lambda_0}^2/4 = 1$, while the density parameter is given by $\Omega_0 = 1 - {\lambda_0}^2/4$ and $\Omega_0 < 1$ (for more detail see [20, 21]).

3.2. Exponential type inflation

Consider a cosmological model in which the contribution of vacuum energy prevails in the total energy density, so that the equation of state $p = -\varrho c^2$ is realized (see, e.g., [22]). In this case we suppose that the scale factor $R(t)$ increases very fast according to the exponential law as in the classical inflation:

$$R(t) = Ae^{Ht}, \tag{27}$$

where A and H are positive constants. Moreover, here H plays the role of the Hubble constant since $\dot{R}/R = H$ for any time. The expression (27) satisfies Eqs. (15) and (16) for $k = 0$ and $k = 1$, but they cannot be satisfied if $k = -1$.

If $k = 0$, a substitution of (27) into (12)–(14) gives the simple solution

$$\sigma = \text{const} , \quad \vartheta = 0 , \quad \varrho = \frac{3H^2}{\kappa_0 c^4} = \text{const} , \tag{28}$$

which coincides with the classical de Sitter solution of general relativity.

If $k = 1$, then from (12) to (14) we find:

$$\sigma(t) = \pm \frac{2c}{\sqrt{3}AH} e^{-Ht} + \sigma_0 , \tag{29}$$

$$\dot{\sigma}(t) = \mp \frac{2c}{\sqrt{3}A} e^{-Ht}, \tag{30}$$

$$\varrho(t) = \frac{3H^2}{\kappa_0 c^4}\left(1 + \frac{2c^2}{3A^2H^2}\, e^{-2Ht}\right), \tag{31}$$

$$\vartheta(t) = \mp\, \frac{2\sqrt{3}\, H}{\kappa_0 c A}\, e^{-Ht}\ . \tag{32}$$

Notice that these solutions asymptotically tend to (28) when t is large. Consequently, with an exponential growth of the scale factor, the σ-field effects become more and more negligible, so that at large t the model passes over to the standard inflationary de-Sitter model. It is interesting to note that within this simplest inflationary model we get a natural explanation of why at present we do not observe scalar field effects or why these effects are so small.

Let us indicate another exponential type of solutions possessing such properties. Such a solution corresponding to expansion without a singularity is given by

$$R(t) = A\cosh \omega t, \tag{33}$$

$$\sigma(t) = \pm\, \frac{2a}{\sqrt{3}\omega}\, \arctan\left(\sinh \omega t\right) + \sigma_0\ , \tag{34}$$

$$\dot{\sigma}(t) = \pm\, \frac{2a}{\sqrt{3}}\, \frac{1}{\cosh \omega t}\ , \tag{35}$$

$$\varrho(t) = \frac{3\omega^2}{\kappa_0 c^4}\left(1 + \frac{2a^2}{3\omega^2}\, \frac{1}{\cosh^2 \omega t}\right)\ , \tag{36}$$

$$\vartheta(t) = \mp\, \frac{2\sqrt{3}a\omega}{\kappa_0 c^2}\, \frac{\sinh \omega t}{\cosh^2 \omega t}\ , \tag{37}$$

where $a \equiv \sqrt{kc^2A^{-2} - \omega^2}$, A and ω are positive constants. In this case the conditions (15) and (16) can only be satisfied if $k = 1$.

Next, one can find a solution corresponding to expansion from a singularity:

$$R(t) = A\sinh \omega t, \tag{38}$$

$$\sigma(t) = \pm\, \frac{a}{\sqrt{3}\omega}\, \ln \frac{\cosh \omega t - 1}{\cosh \omega t + 1} + \sigma_0\ , \tag{39}$$

$$\dot{\sigma}(t) = \pm\, \frac{2a}{\sqrt{3}}\, \frac{1}{\sinh \omega t}\ , \tag{40}$$

$$\varrho(t) = \frac{3\omega^2}{\kappa_0 c^4}\left(1 + \frac{2a^2}{3\omega^2}\, \frac{1}{\sinh^2 \omega t}\right)\ , \tag{41}$$

$$\vartheta(t) = \mp\, \frac{2\sqrt{3}a\omega}{\kappa_0 c^2}\, \frac{\cosh \omega t}{\sinh^2 \omega t}\ , \tag{42}$$

where now $a \equiv \sqrt{kc^2A^{-2} + \omega^2}$. In this case (15) and (16) is automatically true for $k = 1$ and $k = 0$ and can always be satisfied if $k = -1$. It should be noted that there is one more solution, corresponding to harmonic behaviour of the scale factor:

$$R(t) = A\sin \omega t, \tag{43}$$

$$\sigma(t) = \pm \frac{a}{\sqrt{3}\omega} \ln \frac{\cos \omega t - 1}{\cos \omega t + 1} + \sigma_0 \ , \tag{44}$$

$$\dot{\sigma}(t) = \pm \frac{2a}{\sqrt{3}} \frac{1}{\sin \omega t} \ , \tag{45}$$

$$\varrho(t) = \frac{3\omega^2}{\kappa_0 c^4} \left(-1 + \frac{2a^2}{3\omega^2} \frac{1}{\sin^2 \omega t} \right) \ , \tag{46}$$

$$\vartheta(t) = \mp \frac{2\sqrt{3}a\omega}{\kappa_0 c^2} \frac{\cos \omega t}{\sin^2 \omega t} \ . \tag{47}$$

This is the so-called [19] trigonometric counterpart to the solution (38) to (42).

3.3. Power law inflation

As a further example, consider a power-law behaviour of the scale factor:

$$R(t) = At^m \quad (m \geq 1), \tag{48}$$

where A and m are positive constants. The relation (48) always satisfies (15) and (16) for $k = 0$ and for $k = 1$ and can be satisfied for a restricted period of time if $k = -1$.

If $k = 0$, Eqs. (12) to (14) give the simple solution

$$\sigma(t) = \pm\, 2 \sqrt{\frac{m}{3}} \ln t + \sigma_0 \ , \tag{49}$$

$$\dot{\sigma}(t) = \pm\, 2 \sqrt{\frac{m}{3}} \frac{1}{t} \ , \tag{50}$$

$$\varrho(t) = \frac{3m^2 - m}{\kappa_0 c^4} \frac{1}{t^2} \ , \tag{51}$$

$$\vartheta(t) = \mp \frac{\sqrt{3m}(3m-1)}{\kappa_0 c^2} \frac{1}{t^2} \ . \tag{52}$$

In this case the equation of state $\vartheta = \vartheta(\varrho)$ in an explicit form is given by

$$\vartheta = \mp \sqrt{3/m}\, \varrho c^2 \ . \tag{53}$$

If $k = \pm 1$, then we have ($m \neq 1$)

$$\sigma(t) = \pm \frac{\sqrt{m}}{3(1-m)} \left[2\sqrt{1 + at^{-2m+2}} + \ln \frac{\sqrt{1 + at^{-2m+2}} - 1}{\sqrt{1 + at^{-2m+2}} + 1} \right] + \sigma_0 \ , \tag{54}$$

$$\dot{\sigma}(t) = \pm\, 2 \sqrt{\frac{m}{3}} \frac{\sqrt{1 + at^{-2m+2}}}{t} \ , \tag{55}$$

$$\varrho(t) = \frac{1}{\kappa_0 c^4} \frac{3m^2 - m + 2mat^{-2m+2}}{t^2} \ , \tag{56}$$

$$\vartheta(t) = \mp \frac{\sqrt{3m}}{\kappa_0 c^2} \frac{3m - 1 + 2mat^{-2m+2}}{t^2 \sqrt{1 + at^{-2m+2}}} \ , \tag{57}$$

where now $a \equiv kc^2 m^{-1} A^{-2}$. It is easy to see that this solution tends asymptotically to (49)–(52) at large t. Let us also note that this family of solutions is like the standard non-inflationary big-bang if $1/3 \leq m < 1$. In the case $m = 1$ we have linear inflation. The corresponding solution is given by

$$R(t) = At, \tag{58}$$

$$\sigma(t) = \pm\, 2\, \sqrt{(1+a)/3}\, \ln t + \sigma_0 \,, \tag{59}$$

$$\dot{\sigma}(t) = \pm\, 2\, \sqrt{(1+a)/3}\, \frac{1}{t} \,, \tag{60}$$

$$\varrho(t) = \frac{2(1+a)}{\kappa_0 c^4} \frac{1}{t^2} \,, \tag{61}$$

$$\vartheta(t) = \mp \frac{2\, \sqrt{3(1+a)}}{\kappa_0 c^2} \frac{1}{t^2} \,, \tag{62}$$

where now $a \equiv kc^2 A^{-2}$.

4 Conclusions

In the present paper, using the arbitrariness in the choice of an equation of state for the scalaric substrate energy density $\vartheta = \vartheta(\varrho)$ which is not determined within the PUFT, we have proposed an exact solutions generation method for homogeneous and isotropic models of the Universe. This method has allowed us to find new Friedmann-like solutions for the dust model as well as solutions for the simplest inflationary models. It is interesting to note that within the framework of exponential-type of inflation we have obtained a natural explanation of why at present we do not observe σ-field effects or why these effects are so negligible. It should be noted that a cosmological model, in which the σ-field plays an essential role both at early stages of the Universe's evolution and at present, was considered in detail in recent papers by E. Schmutzer [23].

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